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Nonlinear stability analysis of magnetohydrodynamic condensate film flow down a vertical plate with constant heat flux

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Abstract—The generalized kinematic equation for condensate film thickness is taken into consideration at the liquid–vapor interface and is used to investigate the nonlinear stability of film flow down a vertical wall in an applied transverse uniform magnetic field. Results show that both the supercritical stability and the subcritical instability may occur in film flow. The results also indicate that the flow would be stabilized if the mass was transferred into the liquid phase. The supercritical filtered waves are always linearly stable with respect to side-band disturbance. The effect of the magnetic field which can be revealed as the Hartmann number, m, is to stabilize the flow. Therefore, the instability could be counteracted by controlling the applied magnetic field.

1. INTRODUCTION

The instability problem of fluid flow down a vertical or an inclined plate are commonly seen in industrial applications, e.g. in the finishing of painting, the laser cutting process and casting technology. It is known that all macroscopic instability is harmful to fluid flow. Therefore, it would be highly desirable to know the flow configuration and its temporal dependence in order to develop suitable conditions under which the homogeneous growth could be obtained.

The theory for laminar film condensate flow induced by gravity was firstly developed by Nusselt [1] (1961), but the stability of a condensate film had never been investigated until 1970s. Bankoff [2], Marshall and Lee [3] and Lin [4] presented the stability analysis of condensate flow successively. They showed that the critical Reynolds number is small in all practical condensation problems, so the film can be assumed to be unstable. The condensation process has the effect of stabilizing for the film flow. However, the mass transfer due to phase change at the interface was not considered.

Unsal and Thomas [5], Spindler [6] and Kocamustafaogullare [7] investigated stability problem in a more detailed form. Their results point out that condensation has a stabilizing effect but, on the other hand, evaporation has a destabilizing effect. Unsal and Thomas [8] analyzed the nonlinear stability of condensate film flow, but there were some mistakes in the report and only the disturbance of the same mode was considered. The problem of linear stability of condensate film flow with constant wall heat flux was also studied by Marshall and Lee [3] and Spindler [6].

The stability of laminar flows in an applied magnetic field has been studied extensively. Chandrasekhar [9] investigated the stability of flow between coaxial rotating cylinders with a magnetic field added along the axial direction. Stuart [10] studied the stability of pressure flow between parallel plates in a parallel magnetic field and Lock [11] studied the problem with a magnetic field perpendicular to the direction of motion and to the boundary planes. Hsieh [12, 13] has studied the case with the magnetic field perpendicular to an inclined plane. It is found from all these studies that the presence of magnetic fields tends to stabilize the system. In our view, those results of previous studies may have expressed some aspects of the system but did not give a complete picture. Unsal and Thomas [8], Hwang and Weng [14] gave the results of film flow down a vertical plane in more detail, but the magnetic field was not taken into consideration. In this paper, we study the finite-amplitude stability of a film flow down a vertical wall with phase change at the interface and the magnetic field is applied perpendicular to it. The method of multiple scales is applied to solve the nonlinear generalized kinematic equation in a order by order way and obtain a secular equation of Ginzburg–Landau type. Through the nonlinear analysis, we could realize theoretically that the equilibrium finite amplitude, which might be the roughness at the surface, could be controlled by adjusting the magnetic field.

2. GENERALIZED KINEMATIC EQUATION

The governing equations and boundary conditions derived below are based on the formulation of ref.

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NOMENCLATURE

a	r_0/h_0, dimensionless radius distance
B_0	magnetic field strength
C_p	liquid specific heat
d	$d_r + id_i$, complex wave celerity
g	gravitational acceleration
h	film thickness
h_0	local base flow film thickness
h^*	dimensionless thickness
h_{fg}	latent heat
K	liquid thermal conductivity
Nd	$\xi^2/\beta Pr^2$
W	$2^{1/3}\sigma\rho^{-1}v^{-4/3}g^{-1/3}(1-\beta)^{-1/3}$, dimensionless surface tension
P	liquid pressure
P_g	vapor pressure
P^*	dimensionless pressure
Pe	$(Pr\ Re)$, local Peclet number
Pr	$\rho C_p v/K$, Prandtl number
Re	$U_0 h_0/v$, local Reynolds number
Re_c	critical Reynolds number
t	time
t^*	dimensionless time
T	liquid temperature
T_v	vapor saturation temperature
U_0	$g(1-\gamma)h_0^{*2}/2v$, reference velocity
u, v	velocity.

Greek symbols

α	wave number
β	ρ_g/ρ
ξ	$C_p\Delta T/h_{fg}$, heat capacity parameter
η	dimensionless disturbance film thickness
θ	dimensionless temperature
δ	dimensionless disturbance amplitude
v	fluid kinematic velocity
ρ	liquid density
ρ_g	vapor density
σ	surface tension
Ψ	liquid stream function
Ψ^*	dimensionless stream function
λ	disturbance wavelength.

Subscripts

r, z, t	partial differential with respect to subscript.

[14], but being extended to formulate a generalized kinematic equation for the plane in a magnetic field. Consider a layer of an incompressible viscous fluid with phase change at the interface flowing down a vertical plane, as shown in Fig. 1. The governing equations are two-dimensional (2D) mass, momentum and energy conservation equations of the liquid phase. The boundary conditions at wall are taken as the nonslip condition for velocity and a constant heat flux. The boundary conditions at the liquid–vapor interface are considered as the balance of normal and tangential stresses, the relation of interfacial energy balances and the equality of liquid and saturated vapor temperatures.

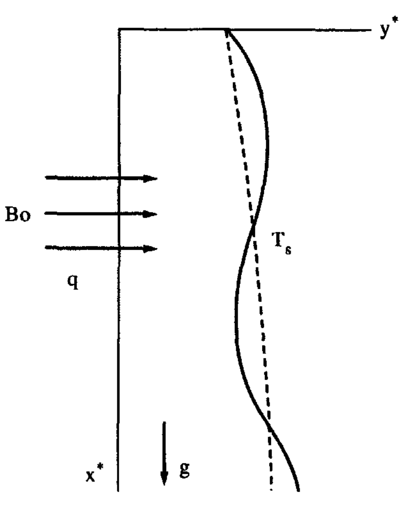

Fig. 1. Physical model and coordinate system.

Let u^* and v^* be the components of the velocity in x^* and y^* directions which is along and is perpendicular to the surface of the plate, respectively. A constant magnetic field of strength B_0 is applied in the positive y-direction. Assuming the magnetic Reynolds number ($Re_m = VL\delta_o\mu_m$) to be very small, which is the case of the most practical applications, the induced magnetic field can be neglected as indicated by Pao [15], where V is characteristic velocity of the fluid, L is a characteristic length, σ_0 is the electric conductivity and μ_m is the magnetic permeability of the fluid. The pondermotive force has one nonvanishing component in the x direction as given in ref. [15] by $F_x = \sigma_o B_o^2 u^*/\rho$, which is opposite to the flow direction.

We assume all physical properties are constant and obtain the equations and boundary conditions as follows:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial x^*} + v\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right) + g - \frac{\sigma_o B_o^2 u^*}{\rho} \tag{2}$$

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial y^*}+v^*\frac{\partial v^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial P^*}{\partial y^*} + v\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right) \quad (3)$$

$$\frac{\partial T}{\partial t^*}+u^*\frac{\partial T}{\partial x^*}+v^*\frac{\partial T}{\partial y^*} = +\frac{K}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^{*2}}+\frac{\partial^2 T^*}{\partial y^{*2}}\right) \quad (4)$$

$$u^* = 0,\quad v^* = 0,\quad k\frac{\partial T}{\partial y^*} = q \quad \text{at} \quad y^* = 0 \quad (5)$$

$$P^*+2\rho v\frac{\partial u^*}{\partial x^*}\left(1+\left(\frac{\partial h^*}{\partial x^*}\right)^2\right)\left(1-\left(\frac{\partial h^{*2}}{\partial x^*}\right)\right)^{-1} + \sigma\frac{\partial^2 h^*}{\partial x^{*2}}\left(1+\left(\frac{\partial h^*}{\partial x^*}\right)^2\right)^{-3/2} + K^2 h_{fg}\rho^{-1}\gamma^{-1}(\gamma-1)\left(\frac{\partial T}{\partial y^*}-\frac{\partial h^*}{\partial x^*}\frac{\partial T^*}{\partial x^*}\right)^2 \times\left(1+\left(\frac{\partial h^*}{\partial x^*}\right)^2\right)^{-1} = P_g \quad (6)$$

$$\frac{\partial u^*}{\partial y^*}+\frac{\partial v^*}{\partial x^*}+4\frac{\partial h^*}{\partial x^*}\frac{\partial u^*}{\partial x^*}\left(\left(\frac{\partial h^*}{\partial x^*}\right)^2-1\right)^{-1} = 0 \quad (7)$$

$$K\left(\frac{\partial T}{\partial y^*}-\frac{\partial h^*}{\partial x^*}\frac{\partial T}{\partial x^*}\right)-\rho h_{fg}\left(\frac{\partial h^*}{\partial t^*}+u^*\frac{\partial h^*}{\partial x^*}-v^*\right) = 0 \quad (8)$$

$$T = T_s \quad \text{at} \quad y^* = h^*. \quad (9)$$

We introduce the stream function ψ^* which is defined as

$$u^* = \frac{\partial \psi^*}{\partial y^*},\quad v^* = -\frac{\partial \psi^*}{\partial x^*}$$

and the following dimensionless quantities:

$$\psi = \frac{\psi^*}{U_0^* h_0^*}$$

$$U_0^* = \frac{g(1-\gamma)h_0^{*2}}{2v}$$

$$\alpha = \frac{2\pi h_0^*}{\lambda}$$

$$P = \frac{(P^*-P_g^*)}{\rho U_0^{*2}}$$

$$h = \frac{h^*}{h_0}$$

$$\theta = \frac{K(T-T_s)}{q h_0^*}$$

$$(x, y, t) = \left(\frac{\alpha x^*}{h_0^*}, \frac{y^*}{h_0^*}, \frac{\alpha t^* U_0^*}{h_0^*}\right)$$

$$Re = \frac{h_0^* U_0^*}{v}$$

$$W = \left(\frac{\sigma^3}{2\rho^3 v^4 g(1-\gamma)}\right)^{1/3}$$

$$\xi = \frac{C_p \Delta T}{h_{fg}}$$

$$Nd = \frac{\xi^2}{\gamma Pr^2}$$

$$Pr = \frac{\rho v C_p}{K}$$

$$Pe = Pr \cdot Re$$

$$m = \left(\frac{\sigma_0 B_0^2 h_0^{*2}}{\rho v}\right)^{1/2}, \quad \text{the Hartmann number.}$$

Then, equations (1)–(9) can be transformed into the following dimensionless form

$$\psi_{yyy}-m^2\psi_y = -2+\alpha Re(P_x+\psi_{yt} + \psi_y\psi_{xy}-\psi_x\psi_{yy})-\alpha^2\psi_{xxy}, \quad (10)$$

$$P_y = -\alpha Re^{-1}\psi_{xyy}+\alpha^2(\psi_y\psi_{xx} - \psi_x\psi_{xy}+\psi_{xt})-\alpha^3 Re^{-1}\psi_{xxx}, \quad (11)$$

$$\theta_{yy} = \alpha Pe(\psi_y\theta_x-\psi_x\theta_y+\theta_t)-\alpha^2\theta_{xx}, \quad (12)$$

$$\psi = \psi_x = \psi_y = 0 \quad \frac{\partial\theta}{\partial y} = 1 \quad \text{at} \quad y = 0 \quad (13)$$

$$P+2\alpha Re^{-1}\psi_{xy}(1+\alpha^2 h_x^2)(1-\alpha^2 h_x^2)^{-1} + 2\alpha^2 W Re^{-5/3} h_{xx}(1+\alpha^2 h_x^2)^{-3/2} + (\gamma-1)N_d Re^{-2}(\theta_y-\alpha^2 h_x\theta_x)^2 \times(1+\alpha^2 h_x^2)^{-1} = 0 \quad (14)$$

$$\psi_{yy} = \alpha^2\psi_{xx}+4\alpha^2\psi_{xy}h_x(1-\alpha^2 h_x^2)^{-1} \quad (15)$$

$$Re^{1/3}\xi(\theta_y-\alpha^2 h_x\theta_x)-\alpha Pe(h_t+\psi_y h_x+\psi_x) = 0 \quad (16)$$

$$\theta = 0 \quad \text{at} \quad y = h. \quad (17)$$

It is noted that equation (17) which is derived from the energy balance, will be used to determine the time evolution of the film thickness, and we call it the generalized kinematic condition.

Since the long wavelength (small wavenumber α) modes are the most unstable ones for film flow, we expand ψ, P and θ in the following form:

$$\left.\begin{aligned}\psi &= \psi_0+\alpha\psi_1+\cdots,\\ P &= P_0+\alpha P_1+\cdots,\\ \theta &= \theta_0+\alpha\theta_1+\cdots,\end{aligned}\right\} \quad (18)$$

The above expression is substituted into equations (10)–(17), and solved order by order. The zeroth and first order solutions are given as follows:

Zeroth order:

$$\psi_0 = \frac{2}{m^3}\tanh(mh)\cosh(my) - \frac{2}{m^3}\sinh(my) + \frac{2}{m^2}y - \frac{2}{m^3}\tanh(mh)$$

$$P_0 = -2\alpha^2 W Re^{-5/3} h_{xx}$$

$$\theta_0 = y - h.$$

First order:

$$\psi_1 = Re\{r_1\cosh(my) + r_2\sinh(my) + r_3 y\cosh(my) + r_4 y\sinh(my) + r_5 y + r_6\}$$

$$P_1 = \frac{-2Re^{-1}h_x\,\mathrm{sech}^2(mh)}{m}[\sinh(my) - \sinh(mh)]$$

$$\theta_1 = Pe\Bigg[-\frac{2}{m^4}\tanh(mh)h^{-2}h_x\sinh(my) + \frac{2}{m^4}h_x\cosh(my) - \frac{1}{m^2}h_x y^2 - \frac{2}{m^4}h_x\,\mathrm{sech}^2(mh)\cosh(my) + \frac{y^2}{m^2}h_x\,\mathrm{sech}^2(mh) - \tfrac{1}{2}h_t y^2 + \frac{2}{m^3}\tanh(mh)y^2 h_x - \frac{2}{m^3}hh_x\tanh(mh) + \tfrac{1}{2}h_t h^2 + \frac{1}{m^2}h^2 h_x\tanh^2(mh)\Bigg].$$

The zeroth- and first-order solutions are then substituted into equation (16) and h_t, which appears in the first-order solution of (16), can be eliminated. This yields the following nonlinear generalized kinematic equation, which is simpler to handle:

$$h_t + X(h) + A(h)h_x + B(h)h_{xx} + C(h)h_{xxxx} + D(h)h_x^2 + E(h)h_x h_{xxx} = 0 \quad (19)$$

where

$$X(h) = -\frac{Re^{1/3}\xi}{\alpha Pe} + \frac{Re^{2/3}\xi 2}{\alpha Pe}h$$

$$A(h) = \frac{2}{m^2}\tanh^2(mh) + \xi\,Re^{1/3}\times\left(\frac{2}{m^3}\mathrm{sech}(mh)\tanh(mh) - \frac{2}{m^3}\tanh(mh)\right) + \frac{\xi\,Re^{1/3}}{Pr}\Bigg(-\frac{2}{m^3}\tanh(mh) + \frac{2}{m^3}\mathrm{sech}(mh)\tanh(mh) - \frac{3}{m^3}\mathrm{sech}^2(mh)\tanh(mh) + \frac{1}{m^2}h\,\mathrm{sech}^4(mh) - \frac{2h}{m^2}\mathrm{sech}^2(mh)\tanh^2(mh)\Bigg)$$

$$B(h) = \alpha\,Re\left(\frac{-6}{m^6}\mathrm{sech}^2(mh)\tanh^2(mh) + \frac{4h}{m^5}\tanh(mh)\,\mathrm{sech}^2(mh) + \frac{2h}{m^5}\mathrm{sech}^4(mh)\tanh(mh)\right)$$

$$C(h) = \frac{2}{m^2}\alpha^3\bar{w}Re^{-2/3}h - \frac{2}{m^3}\alpha^3\bar{w}Re^{-2/3}\tanh(mh)$$

$$D(h) = \frac{2\alpha Re^{2/3}\xi^2 h}{Pe} - \frac{\alpha\xi\,Re^{1/3}}{Pe} + \alpha\,Re\Bigg(\frac{4}{m^5}\tanh(mh)\,\mathrm{sech}^2(mh) + \frac{4h}{m^4}\mathrm{sech}^4(mh) - \frac{8h}{m^4}\tanh^2(mh)\,\mathrm{sech}^2(mh) + \frac{12}{m^5}\mathrm{sech}^2(mh)\tanh^3(mh) - \frac{10}{m^5}\mathrm{sech}^4(mh)\tanh(mh) - \frac{8h}{m^4}\mathrm{sech}^4(mh)\tan^2 h(mh) + \frac{2h}{m^4}\mathrm{sech}^6(mh)\Bigg)$$

$$E(h) = \frac{\tanh^2(mh)}{m^2}2\alpha^3\bar{w}\,Re^{-2/3}.$$

3. STABILITY ANALYSIS

The nondimensional film thickness for the perturbed state may be expanded in the following form:

$$h = 1 + \eta \quad (20)$$

where η is the perturbation of the thickness.

Substituting equations (20) into equation (19), keeping terms up to $O(\eta^3)$, leads to the evolution of η:

$$\eta_1+X\eta+A\eta_x+B\eta_{xx}+C\eta_{xxxx}$$
$$+D\eta_x^2+E\eta_x\eta_{xxx}=-\left[\frac{X''}{2}\eta^2+\frac{X''}{6}\eta^3\right.$$
$$+\left(A'\eta+\frac{A''}{2}\eta^2\right)\eta_x+\left(B''\eta+\frac{B''}{2}\eta^2\right)\eta_{xx}$$
$$+\left(C'\eta+\frac{C''}{2}\eta^2\right)\eta_{xxxx}+(B+B'\eta)\eta_x^2$$
$$\left.+(E+E'\eta)\eta_x\eta_{xx}\right]+O(\eta^4) \qquad (21)$$

where the value of X, A and their derivations are evaluated at $h = 1$.

For the linear stability analysis, we neglect the nonlinear part of (21) and obtain the linearized equation

$$\frac{\partial\eta}{\partial t}+X'\eta+A\frac{\partial\eta}{\partial x}+B\frac{\partial^2\eta}{\partial x^2}+C\frac{\partial^4\eta}{\partial x^4}=0. \qquad (22)$$

Assuming the normal mode solution to be

$$\eta=\Gamma\exp[i(x-\mathrm{d}t)]+\Gamma\exp[-i(x-\mathrm{d}t)] \qquad (23)$$

the complex wave celerity corresponding to the linear stability problem is given by

$$d=d_r+id_i=A+i(B-C-X'). \qquad (24)$$

It is noted that, from the expression for the wave speed d_r, long waves in a liquid film travel at approximately twice the speed of the unperturbed surface. Also $d_i = 0$ gives the neutral stability curve.

For the nonlinear stability analysis, we use the method of multiple scales, according to

$$\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}+\varepsilon\frac{\partial}{\partial t_1}+\varepsilon^2\frac{\partial}{\partial t_2},\quad \frac{\partial}{\partial x}\to\frac{\partial}{\partial x}+\varepsilon\frac{\partial}{\partial x_1},$$
$$\eta(\alpha,x,x_1,t,t_1,t_2)=\varepsilon\eta_1+\varepsilon^2\eta_2+\varepsilon^3\eta_3,$$

where ε is a small parameter, then equation (21) becomes

$$(L_0+\varepsilon L_1+\varepsilon^2L_2)(\varepsilon\eta_1+\varepsilon^2\eta_2+\varepsilon^3\eta_3)=-\varepsilon^2N_2-\varepsilon^3N_3, \qquad (25)$$

where

$$L_0=\frac{\partial}{\partial t}+X+A\frac{\partial}{\partial x}+B\frac{\partial^2}{\partial x^2}+C\frac{\partial^4}{\partial x^4}$$
$$L_1=\frac{\partial}{\partial t_1}+A\frac{\partial}{\partial x_1}+2D\frac{\partial}{\partial x}\frac{\partial}{\partial x_1}+4C\frac{\partial^3}{\partial x^3}\frac{\partial}{\partial x_1}$$
$$L_2=\frac{\partial}{\partial t_2}+B\frac{\partial^2}{\partial x_1^2}+6C\frac{\partial^3}{\partial x^3}\frac{\partial}{\partial x_1}$$
$$N_2=\frac{X''}{2}\eta_1^2+A'\eta_1\eta_2+C'\eta_1\eta_{1xxx}+C'\eta_1\eta_{1xxxx}+D\eta_{1x}^2+E\eta_{1x}\eta_{1xxx}$$

and

$$N_3=X''\eta_1\eta_2+\frac{X''}{6}\eta_1^3+A'(\eta_1\eta_{2x}+\eta_1\eta_{1x_1}+\eta_{1x}\eta_2)+\frac{A''}{2}\eta_1^2\eta_{1x}$$
$$+D'(\eta_{1xx}\eta_2+\eta_1\eta_{2xx}+2\eta_1)+\frac{B''}{2}\eta_1^2\eta\eta_{1xx}+C'(\eta_{xxxx}\eta_2$$
$$+\eta_1\eta_{2xxxx_1}+4\eta_1\eta_{1xxxx_1})+\frac{C''}{2}\eta_1^2\eta_{1xxxx}+D(2\eta_{1x}\eta_{2x}$$
$$+2\eta_{1x_1}\eta_{1x})+D'\eta_1\eta_{1x}^2+E(\eta_{1xxx}\eta_{2x}+\eta_{1xxx}\eta_{1x_1}$$
$$+\eta_{1x}\eta_{xxx_1}+\eta_{1x}\eta_{2xxx})+E'\eta_1\eta_{1x}\eta_{1xxx}.$$

Equation (25) is then solved order by order: The $O(\varepsilon)$ equation is $L_0\eta_1 = 0$, its solution is of the form

$$\eta_1=\Gamma(x_1,t_1,t_2)\exp[i(x-d_rt)]+C\cdot C\cdot \qquad (26)$$

then the solution of η_2 and the secular condition for $O(\varepsilon^3)$ are

$$\eta_2=C_1\Gamma^2\exp[2i(x-d_rt)]+C\cdot C\cdot \qquad (27)$$

and

$$\frac{\partial\Gamma}{\partial t_2}+D_1\frac{\partial^2\Gamma}{\partial x^2}-\varepsilon^{-2}d_i\Gamma+(E_1+iF_1)\Gamma^2\Gamma=0 \qquad (28)$$

respectively, where

$$C_1=C_{1r}+iC_{1i}=(16C-4B+X)^{-1}\times\left(\frac{X'}{2}B'+C'-D+E-iA'\right)$$
$$D_1=B-6C$$
$$E_1=\frac{-X''}{2}+\tfrac{3}{2}C''-\tfrac{1}{2}D'-E'-A'C_{1i}+(X'-5B'+17C'+4D-10E)C_{1r},$$
$$F_1=\frac{A''}{2}+A'C_{1r}+(X'-5B'+17C'+4D-10E)C_{1i}.$$

Equation (28) can be used to study the nonlinear behavior of film flow. For a filtered wave, there is no spatial modulation and the diffusion term in equation (28) vanishes. The solution of this equation may be written as

$$\Gamma_\infty = a \exp(-ibt_2). \tag{29}$$

Substituting equation (29) into equation (28), neglecting the second term, we obtain the following results:

$$\varepsilon a = \left(\frac{d_i}{E_1}\right)^{1/2} \tag{30}$$

and

$$\varepsilon^2 b = F_1\left(\frac{d_i}{E_1}\right). \tag{31}$$

We know from equation (30) that, in the linear unstable region ($d_i > 0$) the condition for the existence of a supercritical wave is $E_1 > 0$ and $2\,\varepsilon\,a$ is just the final amplitude. On the other hand, in the linear stable region ($d_i < 0$) if $E_1 < 0$, then the film flow has the behavior of subcritical instability and $2\varepsilon a$ is the threshold amplitude.

4. RESULTS AND DISCUSSIONS

The linear stability analysis yields the neutral stability curve which is determined by $\alpha d_i = 0$ and separates the α–Re plane into two regions: the linear stable region where small disturbances decay with time; and the linearly unstable region where small disturbances grow with time. For the purpose of numerical calculations, the values of the dimensionless parameters are fixed as $W = 1000$, $\xi = 0.1$ and $Pr = 2.62$. The results are, in general, in agreement with the results of previous studies (Spindler [6]; Hwang and Weng [14]) when $m = 0$.

Figure 2 shows the neutral stability curve for condensate film flow with different Hartmann numbers. The stable region will be expanded when m increases further.

The nonlinear stability analysis is used to study whether the finite-amplitude disturbance in the linear stable region will cause instability (subcritical instability), and to study whether the subsequent nonlinear evolution of disturbance in the linear unstable region will re-develop into a new equilibrium state with a finite amplitude (supercritical stability) or grow to be unstable. By inspection of equation (30), one can find that the negative value of E_1 will make the system unstable. Such kinds of instability in the linear stable region are called subcritical instability; i.e. the disturbance amplitude is larger than the threshold amplitude, then the amplitude will increase although the prediction by linear theory is stable. On the other hand, such instability in the unstable region will cause the system to reach an explosive state which could be considered as the solution of a complex pattern.

The hatched area in Fig. 2 near the neutral stability curve shows that both subcritical instability ($d_i > 0, E_1 < 0$) and the explosive solution ($d_i < 0, E_1 < 0$) are possible for the film flow, the nonlinear critical Reynolds number Re_c is important. If $Re < Re_c$ the film flow is nonlinear stable; otherwise, the film flow is nonlinear unstable near the region of upper neutral curve. As shown in Fig. 2, the increment of the Hartmann number will increase the Re_c, that is, the added magnetic field will stabilize the flow.

It could be shown from the nonlinear instability analysis that the system will be unstable if the initial

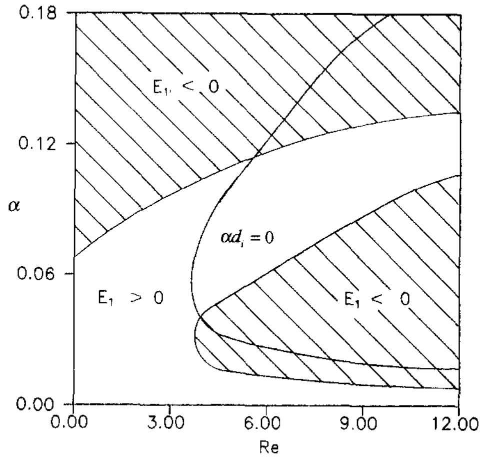

Fig. 2(a). Stability curve of condensate film flow: $m = 0$.

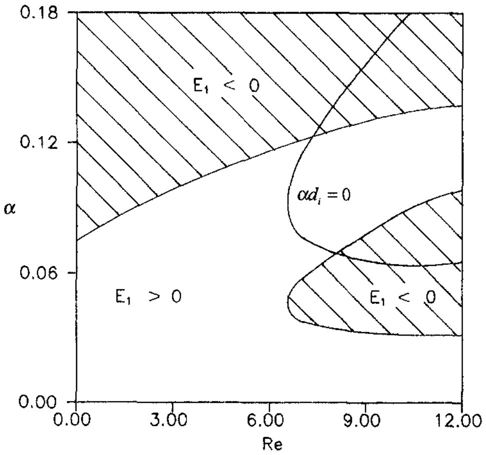

Fig. 2(b). Stability curve of condensate film flow: $m = 0.2$.

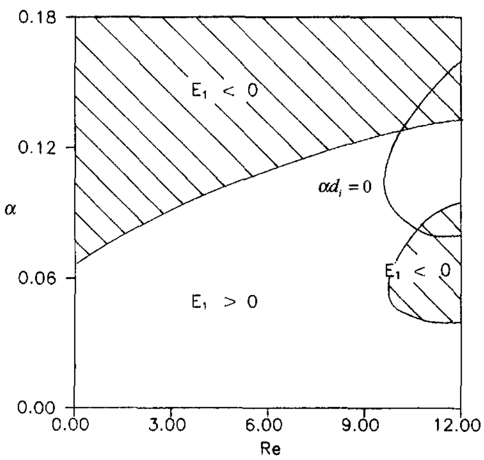

Fig. 2(c). Stability curve of condensate film flow: $m = 0.5$.

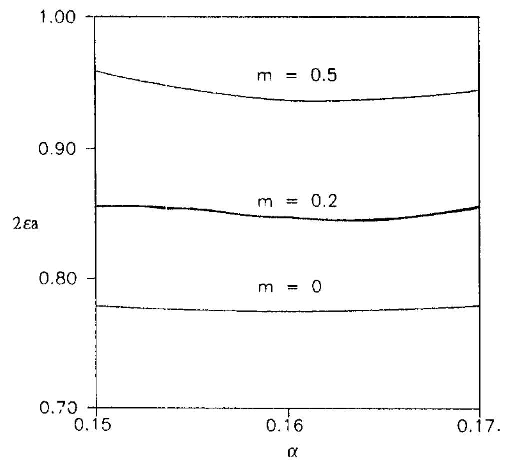

Fig. 3. Threshold amplitude in the subcritical unstable region with different Hartmann number.

finite amplitude disturbance is greater than the threshold amplitude. Figure 3 displays the threshold amplitude in the subcritical unstable region for different Hartmann numbers with the value of $Re = 10$. The decrease of the Hartmann number will lower the threshold amplitude; that is, it will be more stable.

In the linearly unstable region, the linear amplification rate is positive, while the nonlinear amplification rate is negative. Therefore, the linearly infinitesimal disturbance in the unstable region will not grow infinitely, but rather reaches an equilibrium amplitude that is obtained from equation (30). Figure 4 displays the supercritical stable amplitude for different Hartmann numbers with the value of $Re = 10$. It is found that the increase of the Hartmann number will lower the threshold amplitude, and therefore the flow will be more stable.

From the above discussion, the effect of magnetic field will strongly affect the stability characteristics of film flow. The increase in Hartmann number will

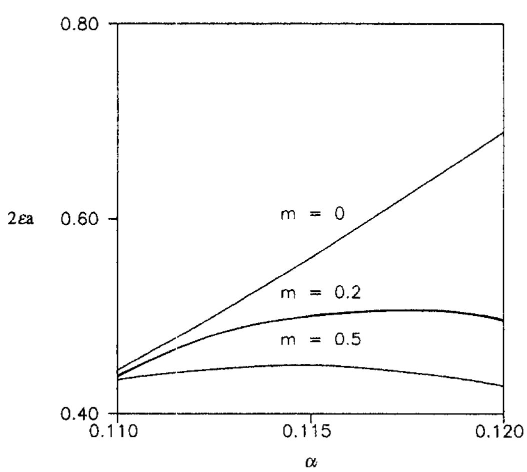

Fig. 4. Threshold amplitude in the supercritical stable region with different Hartmann number.

increase the stability of film flow. Because the magnetic force is applied in the opposite direction to the fluid flow it will cause the flow to be retarded.

5. CONCLUSIONS

The nonlinear instability of a magnetohydrodynamic film flow with phase change at the interface is investigated by the method of perturbation, and a nonlinear generalized kinematic equation is obtained. The phenomena for the magnetohydrodynamic interaction between fluid flow and magnetic field is described in this study.

Linear stability analysis is studied first. The critical Reynolds number could be obtained. The increase of Hartmann number will increase the critical Reynolds number, and so the flow will be more stable. The linear stability analysis only gives the statements of qualitative tendency about the dynamic behavior of film flow, it could not give any statement about the finite amplitude of the disturbed surface, which is more important for the determination of the roughness at the surface. The stationary finite amplitude could be obtained only from the nonlinear stability analysis of the flow.

The method of multiple scale is used for the nonlinear stability analysis. It indicates that there exists supercritical stability in the linear unstable region, and infinitesimal disturbance will develop into a new equilibrium finite amplitude. However, there exists subcritical instability in the linear stable region. The increase of Hartmann number will increase the critical amplitude in the subcritical unstable region and will reduce the amplitude of supercritical stable wave. Therefore, the effect of magnetic force will strongly affect the stability characteristics of film flow. The increase in Hartmann number will increase the stability of film flow.

To stabilize the fluid flow by applying a magnetic field has the advantage for neither electrical nor mechanical contacts with the fluid, which could be of particular importance to the high temperature molten flows in the cases of laser cutting or semiconductor crystal growth. It would be of use in actively controlling a technological process based on the magnetic field, for example, in laser cutting process, where the wave surface is present globally at the molten interface. By applying a magnetic field to counteract the inertia force, the instability could be impeded and the smooth flow maintained. Therefore, it can be concluded that increasing the stability of film flow by controlling magnetic field, a film flow with optimum conditions could be obtained.

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